MCA-I / SEM-II / OPTIMIZATION TECHNIQUES (2024-25)

COMPREHENSIVE CONCURRENT EVALUATION 2024-25

Course Code: MT 21

Course Name: Optimization Techniques

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The course is evaluated on the basis of two components

Sr. No.	Parameter / Component	Marks	Date
1	Written Home Assignment on Sequencing (10 Marks) Decision Theory (10 Marks) Game Theory (10 Marks) Markov Chain (10 Marks) Simulation (10 Marks) CPM & PERT (15 Marks) Queuing Theory (10 Marks) Transportation Problem (10 Marks) LPP (15 Marks)	100	As & when communicated
2	Class Test	25	

Assignment on Decision Theory

Q 1) Consider the following pay-off (profit) matrix Action States

States

Action	S1	S2	S3	S4
A1	5	10	18	25
A2	8	7	8	23
A3	21	18	12	21
A4	30	22	19	15

Determine best action using 1) maximin 2) maximax 3) Laplace 4) Hurwicz $\alpha = 0.6$ 5) Minimax Regret principle.

Q 2) A farmer wants to decide which of the three crops he should plant on his 100-acre farm. The profit from each is dependent on the rainfall during the growing season. The farmer has categorized the amount of rainfall as high medium and low. His estimated profit for each is shown in the table.

Rainfall	**Estimated Conditional Profit (Rs)**		
	Crop A	Crop B	Crop C
High	8000	3500	5000
Medium	4500	4500	5000
Low	2000	5000	4000

If the farmer wishes to plant only crop, decide which should be his best crop using 1) maximin 2) maximax 3) Laplace 4) Hurwicz $\alpha = 0.7$ 5) Minimax Regret

Q3) The research department of Hindustan Ltd. has recommended to pay marketing department to launch a shampoo of three different types. The marketing types of shampoo to be launched under the following estimated pay-offs for various level of sales.

Types of Shampoo	**Estimated Sales (in Units)**		
	15000	10000	5000
Egg Shampoo	30	10	10
Clinic Shampoo	40	15	5
Deluxe Shampoo	55	20	3

What will be the marketing manager's decision if 1) maximin 2) maximax 3) Laplace 4) Hurwicz $\alpha = 0.7$ 5) Minimax Regret principle used.

Q 4) An agricultural company wants to decide which commodity should stock to get maximum profit. It was supplied with the following information. The probability that the monsoon will be excess, normal and deficient is 0.40, 0.30 and 0.30 . The estimated profit or loss three commodities in respect of these different kinds of monsoon are:

Profit per 1 ton			
Monsoon	Excess	Normal	Deficient
Rice	10,000	-4,000	15,000
Wheat	4,000	-3,000	8,000
Maize	4,000	1,000	-1,000

Determine the optimal decision under each of the following decision criteria and show how you arrived at it:

(a) Maximax, (b) Maximin, (c) Minimax regret (savage criterion), (d) Equal likelihood (Laplace), (e) Hurwicz Alpha criterion α=0.8, (f)EMV Find EVPI .

Q5) The payoffs (in Rs) of three Acts A_1, A_2 and A_3 and the possible states of nature S_1,S_2 and S_3 are given below:

State of Nature	**Acts**		
	A1	A2	A3
S1	-20	-50	200
S2	200	-100	-50
S3	400	600	300

The probabilities of the states of nature are 0.3, 0.4 and 0.3 respectively. Determine theoptimal act using EMV. Determine EVPI

Q 6) A retailer purchases berries every morning at Rs. 50 a case and sells for Rs. 80 a case. Any case remaining unsold at the end of the day can be disposed of the next day at a salvage value of Rs. 20 per case (thereafter they have no value). Past sales have ranged from 15 to 18 cases per day. The following is the record of sales for the past 120 days:

No. of cases sold:	15	16	17	18
No. of days:	12	24	48	36

Find how many cases the retailer should purchase per day to maximize his profit?

Q 7) A grocery receives its weekly supply of eggs every Thursday morning. This shipment must last until the following Thursday when a new shipment is received. Any eggs left unsold by Thursday are destroyed. Eggs sell for 520 per hundred and cost 450 per hundred. The weekly demand for eggs at this grocery varies from week to week. From past experience, the following probability distribution is assigned to weekly demand:

Demand
(Hundreds of eggs): 10 11 12 13 14
Probability: 0.1 0.2 0.4 0.2 0.1

This pattern of demand remains stable throughout the year the demand for eggs is not seasonal, and the trend is flat. The problem is: How many eggs should be ordered for delivery every Thursday?

Q 8) Suresh find the probability of demand distribution of luxury cars as follows:

Probability of Demand	0.2	0.4	0.1	0.3
Demand for each car each day	1	2	3	4

The selling price of the car is Rs. 10,00,000/- and it costs to Suresh Rs.6,00,000/- Suresh has always followed a rule for initial purchase, purchase 3 cars. Find the expected daily profit under the decision rule of buying three cars each morning. If the fees for perfect information is Rs. 1,00,000/- calculate the expected monetary value of the venture with perfect information (EVPI)

Assignment on Game Theory

Q 1) Solve the game whose payoff matrix is

		Player B				
Player A		Give Coupons I	Decrease Price II	Maintain Present Strategy III	Increase Advertising IV	Expand the Business
	Give Coupons I	-2	0	0	5	3
	Decrease Price II	3	2	1	2	2
	Maintain Present Strategy III	-4	-3	0	-2	6
	Increase Advertising IV	5	3	-4	2	-6

Q 2) Solve the game whose payoff matrix is

Player A		B1	B2	B3	B4
	A1	1	7	3	4
	A2	5	6	4	5
	A3	7	2	0	3

Q 3) Solve the following game whose payoff matrix is:

	I	II	III	IV	V	VI
I	4	2	0	2	1	1
II	4	3	2	3	2	2
III	4	3	7	-5	1	2
IV	4	3	4	-1	2	2
V	4	3	3	-2	2	2

Q 4) Solve the following game.

	B1	B2
A1	3	-4
A2	-3	4

Q 5) Two firms are competing for business under the similar condition, so thet one firm's gain is other company's loss . Firm A's pay-off matrix is given below

Firm B

	No Ad B1	Medium Ad B2	Heavy Ad B3
No Ad A1	9	8	-7
Medium Ad A2	3	-6	4
Heavy Ad A3	6	7	7

Suggest the optimum strategies for the two firms and net outcome thereof.

Q 6) Find optimal strategies for two companies competing each other on the basis of following pay-off matrix showing gain or loss of customers

Company A	Company B		
	Quantity Discounts	CSR Initiatives	Event sponsorship
Quantity Discounts	20	60	30
CSR Initiatives	-10	30	25
Event sponsorship	40	50	-30

Suggest the optimum strategies for the two firms and net outcome thereof.

Assignment on Markov Chain

Q 1) Consider market share of e-wallet companies, Company A – 30 %, Company B – 35 %, Company C – 35 % respectively. Their transition probability matrix be

	A	B	C
A	0.70	0.10	0.2
B	0.30	0.30	0.40
C	0.20	0.30	0.50

Determine market share of e-wallet companies after one year and 2 year.

Q 2) Due to high level of employment in Pune, mobility of population towards Pune in last decade has increased tremendously. This mobility is from other parts of Maharashtra State. Following table shows mobility of population amongst Pune, Kolhapur, Nagpur in percentages.
Assume population proportion is 70,00000, 20,00000 and 10,00000 respectively.

	Pune	Kolhapur	Nagpur
Pune	50 %	30 %	20 %
Kolhapur	10 %	70 %	20 %
Nagpur	10 %	40 %	50 %

Assume mobility period is 10 yrs. What will be population after 10 yrs. and 20 yrs?

Q 3) Three brands of product P, Q and R are having market share as 30%, 30% and 40% respectively.
Customers shift their brands. Brand switching matrix every quarter is given below.

P	50%	30%	20%
Q	20%	70%	10%
R	20%	20%	60%

Find market share at the end of quarter.

Q 4) The leading brewery on the West Coast (labeled A) has hired an OR analyst to analyze its market position. It is particularly concerned about its major competitor (labeled B). The analyst believes that brand switching can be modeled as a Markov chain using three states, with states A and B representing customers drinking beer produced from the aforementioned breweries and state C representing all other brands. Data are taken monthly, and the analyst has constructed the following (one-step) transition matrix from past data.

	A	B	C
A	0.7	0.2	0.1
B	0.2	0.75	0.05
C	0.1	0.1	0.8

What are the steady-state market shares for the two major breweries?

Q 5) A grocery stocks his store with three types of detergents A, B and C. When brand A is sold out the probability is 0.7 that he stocks up with brand A again. When he sells out brand B the probability is 0.8 that he will stock up again with brand B. Finally, when he sells out brand C the probability is 0.6 that he will stock up with brand C again. When he switches to another detergent he does so with equal probability for the remaining two brands. Find the transition matrix. In the long run, how does he stock up with detergents?

Q 6) The XYZ company is planning an extensive advertising campaign to increase the company's market share. The Company is faced with the job of choosing between the two campaigns that have been recommended. It has decided to test each proposal in two test areas where the initial market shares of the competing firms and the initial transition probability matrices are the same. Also, the market shares of the firms are close to their national average, which are : brand X (or XYZ Co.), 28 per cent; brand A, 39 percent ; and brand B, 33 per cent. The matrix of initial transition probabilities for both areas is :

	Brand X	Brand A	Brand B
Brand X	0.6	0.3	0.1
Brand A	0.2	0.7	0.1
Brand B	0.1	0.1	0.8

Find market share after one and two period.

Q 7) A market survey is made on three brand of breakfast food X, Y and Z. Every time the customer purchases anew package, he may buy the same brand or switch to another brand. The following estimates are obtained, expressed as decimal fractions:

Present brand Brand Just purchased

	X	Y	Z
X	0.7	0.2	0.1
Y	0.2	0.7	0.1
Z	0.1	0.2	0.7

At this time it is estimated that 30 per cent of the people buy brand X, 20 per cent brand Y and 50 per cent brand Z. What will the distribution of customers be two time period late and at equilibrium?

Topic: Simulation

Q 1) Following is the distribution of defective pieces in a manufacturing process of a MNC in Pune.

No. of Defective	0	10	20	30	40	50
Probability	0.01	0.20	0.15	0.50	0.12	0.02

Consider the following sequence of random numbers

R No. 68, 25, 39, 41, 66, 87, 45, 34, 18, 09

Using this sequence. Simulate the number of defective items for next 10 days.

Q 2) In a cricket season for a one day match a bowler bowled 50 balls. The frequency distribution of runs scored per ball is as given below:

Runs / ball	0	1	2	3	4	5	6
Number of balls	15	10	10	4	8	2	1

Simulate the system for 2 overs and find average runs given in 2 overs by him. Use the following random numbers: 78, 23, 15, 09, 38, 18, 75, 29, 35, 27, 77, and 52. Also find how many times no run /ball.

Q 3) For a single channel queuing model the data about the inter - arrival time of the workers at a tools – crib for collecting the tools and the service time required by the attendant at the tool crib is as follow.

Inter Arrival Time		Service Time	
Time (Minutes)	Frequency	Time (in minutes)	Probability
2	10	1	0.05
4	6	2	0.20
6	2	3	0.50
8	2	4	0.20
		5	0.05

Simulate the system for the net 5 arrival and compute the following.
a. Percentage of time the attendant is idle b. Average waiting time for the workers at the tool crib.

Use the random number
10 21 56 75 47 for inter arrival time
65 59 02 71 26 for service time.

Q 4) An engineering firm utilizes re- order level system to replenish stock based on average demand. The demand is given below.

Demand Per Week	0	1	2	3	4	5	6
Frequency	2	8	22	34	18	9	7

Generate the demand for next 20 weeks using the random numbers given below and calculate average demand. 68, 46, 87, 32, 78, 72, 27, 60, 06, 40, 83, 39, 97, 11, 06, 77, 49, 31, 71, and 92

Q 5) A bakery keeps a stock of popular brand of cake. Daily demand based on past experience is given below.

Daily Demand	0	10	20	30	40	50
Probability	0.01	0.15	0.20	0.50	0.12	0.02

Using the following random numbers simulate the demand for next 10 days and also calculate the average demand for the cake basis of simulated data. Random numbers: 45, 72, 56, 51, 79, 9, 61, 43, 31 and 81

Q 6) Haggins plumbing and heating maintains a stock of 30 gallon hot water heaters that it sells to home owners and installs for them. The owner likes the idea of having large supply on hand so as to meet all customer demand, but he also recognizes that it is expensive to do so. He examine hot heater sales over the past 50 weeks and notes the following

Hot water heater sales per week	Number of weeks this number was sold
4	6
5	5
6	9
7	12
8	8
9	7
10	3
Total	**50**

Using the random numbers given below, simulate demand for 10 weeks and answer the following questions.

(i) If stock of 8 hot water heaters is maintained how many times will the company be stock out in 10 weeks.

(ii) What is the average number of heater demand per week.

Random numbers : 10, 24, 03, 32, 23, 59, 95, 34, 34, 51

Q 7) Following is the distribution of defective pieces in a manufacturing process of a MNC in Pune. Consider the following sequence of random numbers
RNo.48 78, 19, 51, 56, 77, 15, 14, 68, 09
Using this sequence. Simulate the number of defective items for next 10 days. Find average defective items.

No. of Defective items	0	10	20	30	40	50
Probability	0.01	0.20	0.15	0.50	0.12	0.02

Q 8) Following data is related to frequency of student absenteeism in a MBA Class.

Generate the students absenteeism for next 10 weeks. Also find out average absenteeism. Use the following random number
87, 05, 35, 43, 77, 61, 09, 65, 58, 85

No. of Students absent	0	5	10	15	20	25
Frequency	4	22	16	42	10	6

Q 9) A small retailer has studied the weekly receipts and payments over the past 100 weeks and has developed the following set of information:

Using the following sequence of random numbers. Simulate the weekly pattern of receipts and payments for the 10 weeks,, assuming further the beginning bank balance is Rs. 10,000. What is estimated balance at the end of the 10 weekly period ? What is the highest weekly balance during the quarter ? What is the average weekly balance for the quarter.

Weekly Receipts (Rs.)	Probability	Weekly payment (Rs.)	Probability
3,000	0.20	4,000	0.30
5,000	0.30	6,000	0.40
7,000	0.40	8,000	0.20
12,000	0.10	10,000	0.10

Random numbers

For receipts : 03, 91, 38, 55, 17, 46, 32, 42, 69, 72

For payment : 61, 96, 30, 32, 03, 88, 48, 28, 88, 18

Q 10) Dr. Sarang is a dentist who schedules all his patients for 30- minutes appointment some of the patients take more or less than 30 minutes depending on the type of dental work to be done. The following summary shows the various categories of work, their probabilities and the time actually needed to complete the work

Category	Time Required	Prob. of Category
Filling	45 minutes	0.40
Crown	60 minutes	0.15
Cleaning	15 minutes	0.15
Extraction	45 minutes	0.10
Check – up	15 minutes	0.20

Simulate the dentist's clinic for four hours and determine average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients show up at the clinic at exactly their scheduled arrival time starting at 8 a.m.

Use the following random number: 40, 82, 11, 34, 25, 66, 17, 79.

Assignment on CPM and PERT

Q 1) The following information is gathered for a project :

Activity	Preceding Activity	Duration (days)
A	-	1
B	A	3
C	A	4
D	A	3
E	D	2
F	B,C,E	4
G	D	9
H	D	5
I	H	2
J	F,G,I	2

a) Draw the Network Diagram.

b) Determine the Critical Path and Project Duration.

Q 2) The following information is gathered for a project :

Activity	Preceding Activity	Duration (days)
A	-	3
B	-	5
C	B	3
D	A,C	4
E	D	8
F	C	2
G	F	4
H	F	2
I	B	5
J	H,E,G	3

a) Draw the Network Diagram.

b) Determine the Critical Path and Project Duration.

c) Determine earliest and latest start and finish time

Q 3) The following are the time estimates and the precedence relationships of the activities in a project network:

Activity	IMMEDIATE Predecessor Activity	time estimate (weeks)
A	-	4
B	-	7
C	-	3
D	A	6
E	B	4
F	B	7
G	C	6
H	E	10
I	D	3
J	F, G	4
K	H, I	2

Draw the project network diagram. Determine the critical path and the project completion time.

Q 4) Find out the time required to complete the following project and the critical activities

Activity	Predecessor Activity	Optimistic time estimate (to days)	Most likely time estimate (tm days)	Pessimistic time estimate (tp days)
A	-	2	4	6
B	A	3	6	9
C	A	8	10	12
D	B	9	12	15
E	C	8	9	10
F	D, E	16	21	26
G	D, E	19	22	25
H	F	2	5	8
I	G	1	3	5

Also find variance of each activity and variance of project.

Q 5) Consider a project with the following details:

Name of Activity	Predecessor Activity	Duration (Weeks)
A	-	8
B	A	13
C	A	9
D	A	12
E	B	14
F	B	8
G	D	7
H	C, F, G	12
I	C, F, G	9
J	E, H	10
K	I, J	7

Determine the earliest and latest times, the total float for each activity, the critical activities, the slacks of the events and the project completion time.

Q 6) Find out the time, variance and standard deviation of the project with the following time estimates in weeks:

Activity	Optimistic time estimate (to)	Most likely time estimate (tm)	Pessimistic time estimate (tp)
1-2	3	6	9
1-6	2	5	8
2-3	6	12	18
2-4	4	5	6
3-5	8	11	14
4-5	3	7	11
6-7	3	9	15
5-8	2	4	6
7-8	8	16	18

Q 7) Develop a network diagram for the project specified below:

Activity	Immediate Predecessor Activity
A	-
B	A
C, D	B
E	C
F	D
G	E, F

Q 8) Given is the following information regarding a project.

Activity	Preceding Activity	Duration (Days)
A	-	3
B	-	4
C	-	2
D	A,B	5
E	B	1
F	B	3
G	F, C	6
H	B	4
I	E, H	4
J	E, H	2
K	D, J	1
L	K	5

(a) Draw the network for the above project.

(b) Determine the critical path and duration of the project.

Q 9) For the PERT problem find the critical path and project duration. What is variance of project?

Activity	Predecessor	Time		
		Optimistic	Most likely	Pessimistic
A	-	2	5	14
B	-	1	10	12
C	A	0	0	6
D	A	1	4	7
E	C	3	10	15
F	D	3	5	7
G	B	1	2	3
H	E,F	5	10	15
I	G	3	6	9

Q 10)

1) Write short notes on :
(a) Concept of PERT and CPM (b) Concept of Network diagram with example (c) Dummy activities and events with example d) Earliest and latest time with example e) Slack of activity

2) Write a difference between CPM and PERT

Assignment on Queuing Theory

Q 1) At a certain petrol pump. Customers arrive in a Poisson process with an average time of five minutes between successive arrivals. The time taken at the petrol pump to serve customers follow exponential distribution with an average of two minutes find.
i. Arrival and service rate.
ii. The utilization factor
iii. Prob. That there shall be four customers in the system
iv. Prob. That there are more than four customers in the system.
v. Expected queue length
vi. Expected number of customers in the system
vii. Expected time that a customer has to wait in the system
viii. Expected time that a customer has to wait in the queue

Q 2) A repair shop attended to by a single mechanic has an average of four customers an hour who bring small appliances for repair. The mechanic inspects them for defects and quite often can fix them right away or otherwise render diagnosis. This taken him six minutes, on the average, arrivals are Poisson and service time has exponential distribution.
a. Find the proportion of time during which the shop is empty.
b. Find the probability of at least one customers in the shop.
c. What is average no. of customers in the system?
d. Find the average time spent including service.

Q 3) A firm has several machines and wants to install its own service facility for the repair of its machines. The average breakdown rate of machines is 3 per day. The repair time has exponential distribution . The loss incurred due to the lost time of an inoperative machine is Rs. 40 per day. There are two repair facilities available. Facility A has an instillation cost Rs. 20,000 & facility B has Rs. 40,000. With facility A, the total labour cost is Rs. 5,000 per year is with facility B the
Total labor cost Rs. 8000 per year. Facility A can repair 4 machines per day. Facility B can repair 5 machines per day. Both facilities have a life of 4 years. Which facility should be installed?

Q 4) A departmental store has a single cashier. During the rush hours, customers
arrive at a rate of 20 customers per hour. The cashier takes on an average 2.5 minutes per customer for processing.

i) What is the probability the cashier is idle?
ii) What is the average number of customers in the queuing system?
iii) What is average queue length?
iv) What is the average time spent by a customer in the system?

Assignment on LPP

Q 1) Solve the following LPP and interpret the result.

Max. $Z = 8X1 + 16\ X2$
Subject to:
$X1 + X2 \leq 200$
$X2 \leq 125$
$3X1 + 6X2 \leq 900$
$X1, X2 \geq 0$

Q 2) A construction company wants to transfer the construction materials from its warehouse to the project site, using two special type of trucks. A and B. Truck A can carry 1 to one of cement and 4 tons of steel on one trip and truck B carries 2 tons of cement and three tons of steel at a time. The constriction work needs 6 tons of cement and 12 tons of steel daily. Due to limited no. of drivers, only 10 trips can be made to the site daily. The cost of one trip of A and B is Rs. 500 and Rs. 700 respectively how many trips for each truck be planned daily, so as to minimize the transportation cost. Formulate the LPP and solve

Q 3) A firm produces two products P and Q. Daily production upper limit is 600 units for total Production. But at least 300 total units must be produced every day. Machine hours Consumption per unit is 6 for P and 2 for Q. At least 1200 machine hours must be used daily. Manufacturing costs per unit are Rs. 50 for P and Rs. 20 for Q. Find optimal solution for the LPP .

Q 4) Solve the following linear programming problem :
Minimize $Z = 200x + 500y$
Subject to the constraints:
$x + 2y \geq 10$
$3x + 4y \leq 24$
$x \geq 0, y \geq 0$

Q 5) Minimize $Z = 3x + 2y$

Subject to the constraints:

$x + y \geq 8$
$3x + 5y \leq 15$
$x \geq 0, y \geq 0$

Q 6) A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs17.50 per package on nuts and Rs 7.00 per package on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours a day?

Assignment on Transportation Problem

Q 1) Find an initial basic feasible solution for given transportation problem by using
(a) North-West corner method
(b) Least cost method
(c) Vogel's approximation method

	D_1	D_2	D_3	D_4	Supply
S_1	11	13	17	14	250
S_2	16	18	14	10	300
S_3	21	24	13	10	400
Demand	200	225	275	250	

Q 2) A Company has 3 production facilities S1, S2 and S3 with production capacity of 7, 9 and 18 units (in 100's) per week of a product, respectively. These units are to be shipped to 4 warehouses D1, D2, D3 and D4 with requirement of 5, 6, 7 and 14 units (in 100's) per week, respectively. The transportation costs (in rupees) per unit between factories to warehouses are given in the table below.

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

Obtain an optimal solution by MODI method
if the object is to minimize the total transportation cost.

Q 3) A company has factories at F1, F2 and F3 which supply to warehouses at W1, W2 and W3. Weekly factory capacities are 200, 160 and 90 units, respectively. Weekly warehouse requiremnet are 180, 120 and 150 units, respectively. Unit shipping costs (in rupees) are as follows:

	W_1	W_2	W_3	Supply
F_1	16	20	12	200
F_2	14	8	18	160
F_3	26	24	16	90
Demand	180	120	150	450

Determine the optimal distribution for this company to minimize total shipping cost.

Q 4) Find optimal Solution for the following transportation problem

	D1	D2	D3	Supply
S1	4	8	8	76
S2	16	24	16	82
S3	8	16	24	77
Demand	72	102	41	